Predictions from the finite difference solutions are no better than those from the simple flat-plate type methods of Van Driest II,[13] Moore,[17] and Coles.[16] Of course, the use of these latter methods depends on knowing the values of R_{e_θ} which must be obtained by some other means. If possible, experimental profile data should be used for this purpose; however, integral methods may be used since predicted values of $R_{e\theta}$ at the nozzle exit depend mainly on the pressure gradient history rather than the skin-friction law.

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Aerodynamic Characteristics of Two-Dimensional Waverider Configurations

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MANNED shuttle flights into near space will necessitate increased maneuverability and performance of the vehicle that re-enters the upper atmosphere. The overriding aerodynamic parameter in such operations has long been known to be the hypersonic ligt-to-drag ratio (L/D).[1–3]

At high re-entry velocities, marked decreases in entry corridors, landing sites, and mid-course corrections occur. It becomes desirable therefore, to investigate vehicles whose L/D ratios provide a mission with more operational flexibility. Consideration of all these factors have led to the study of advanced Earth entry vehicles of relatively simple shapes having L/D's greater than Apollo's. One such study has led to the Waverider concept. This concept grew out of the need to consider new vehicle designs with high L/D's but whose generated flowfields were simple enough to be mathematically tractable. Since at hypersonic velocities the pressure distribution on the lower surface dominates the lifting characteristics of the wing, designs of flat-top wings were the first to be investigated. T. Nonweiler[4] was the first to consider the concept of inscribing a three-dimensional body within a two-dimensional body-shock configuration. Nonweiler's method of construction consisted of a two-dimensional wedge whose upper surface was colinear to the freestream velocity vector and whose lower surface was inclined at some angle θ_w with respect to the freestream velocity vector. This wedge generates a shock whose angle could easily be calculated from inviscid shock theory. If one now constructs a pair of swept leading edges in the shock plane from the vertex of the wedge, a three-dimensional body is formed (Fig. 1). This body, at the design freestream Mach number, will carry beneath it, in the plane of its swept leading edges, a planar trangular shock. Since the flowfield between the triangular shock and the body is the same as that between the original two-dimensional wedge and its shock, the flow characteristics of this region are immediately known from simple wedge theory. Once the flowfield is known, the lift and drag coefficients of the three-dimensional body due to pressure are calculable, keeping in mind the initial premise; that is, the lower surface dominates the aerodynamic characteristics of the body.

Two disadvantages become immediately apparent: 1) due to its construction, the Waverider is a blunt-based body, and 2) the Waverider's unusual shape (volume distribution and negative dihedral) makes it questionable from a practical engineering design viewpoint. However, both these disadvantages, while contributory to its over-all value as a re-entry vehicle, will not be discussed in this Note nor their subsequent consequences considered in the present analytical treatment.

Previous theoretical treatments of this problem and extensions to axisymmetric configurations are described in Ref. 5–8. Also presented therein are experimental investigations of Waverider designs in both the supersonic and hypersonic regimes. Restricting the present discussion to two-dimensional waveriders, K. Kipke[8] describes the current procedures of analyzing in-

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Index categories: Entry Vehicle Mission Studies and Flight Mechanics; Supersonic and Hypersonic Flow.

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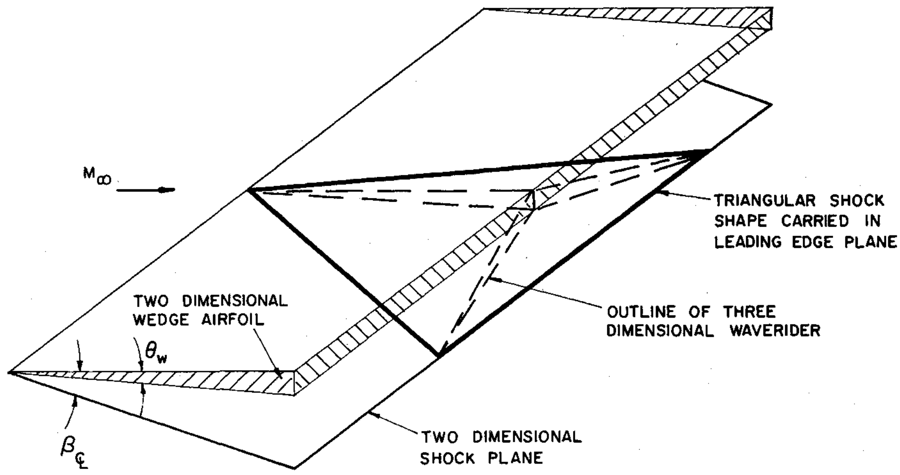

Fig. 1 Waverider construction.

verted V-wings, namely, calculating an inviscid pressure level based upon the centerline flow deflection. This pressure level may be obtained in several ways, for example, simple wedge theory, hypersonic small disturbance theory or a Newtonian approximation. In any event, once it is found, all of the aeromechanic parameters may be obtained since the shock layer flowfield is strictly two-dimensional. Kipke further shows that as the Mach number increases and the Reynolds number decreases, viscous-inviscid interaction effects become increasingly important and, as expected, exert a considerable influence upon the predicted lift-to-drag ratio, pressure distribution, and shock wave location, thereby tending to negate the value of the inviscid design procedures.

In order to circumvent these difficulties, the present author introduces a different inviscid approach based upon an equivalent wedge shape and a viscous correction coefficient utilizing a reference enthalpy technique. In effect, elementary supersonic flow theory is applied to a body-shock configuration viewed in a plane normal to the swept leading edges. It is found that this approach extends the range of applicability of the inviscid design procedure to include higher Mach numbers and lower Reynolds numbers (toward the strong interaction regime) until the viscous-inviscid interaction becomes so great that a more sophisticated analysis is required.

Briefly, the present analysis employs a vector geometry analysis to describe the body whereby the three-dimensional geometry is transformed into an equivalent, two-dimensional, flat surface. This procedure was adopted due to the complications and visual complexities of the waverider design. The upper surface was not considered important in the pressure calculation, however, it was included in the skin-friction coefficient. Again, no correction was taken into account for the blunt-based nature of the design. In order to proceed with the calculation, one need only to specify the basic geometry of the waverider and the free-stream conditions, namely M_∞, α, β_L, L, and $\varphi_o/2$ (Fig. 2). Knowing these quantities, the pertinent aerodynamic parameters such as the semispan, aspect ratio, dehedral angle, etc. may be obtained. In addition, one is able to define a volume parameter,

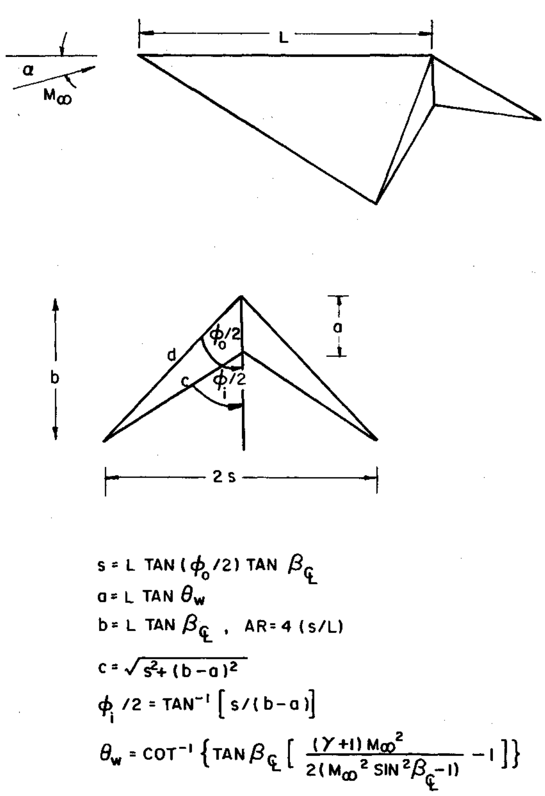

Fig. 2 Waverider geometry.

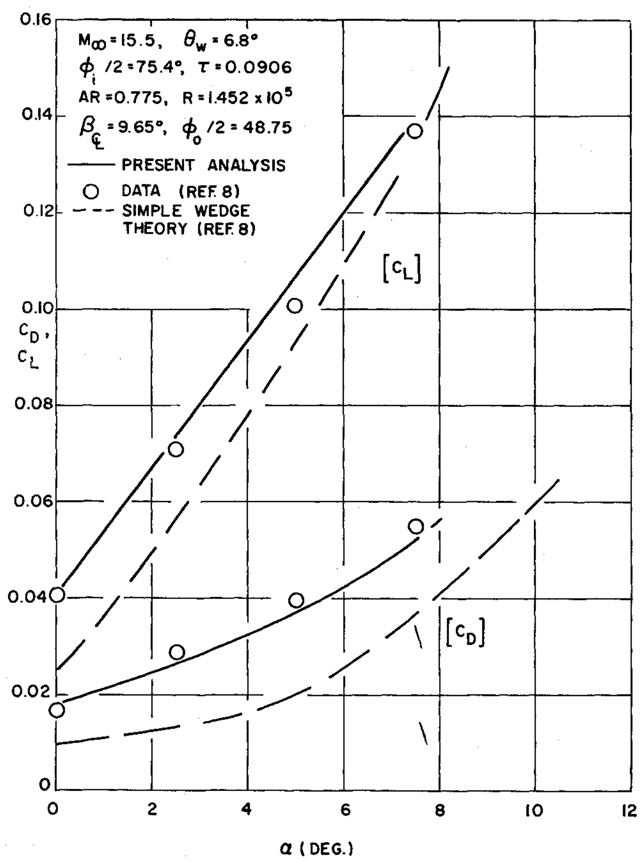

Fig. 3 Lift and drag coefficient vs angle of attack.

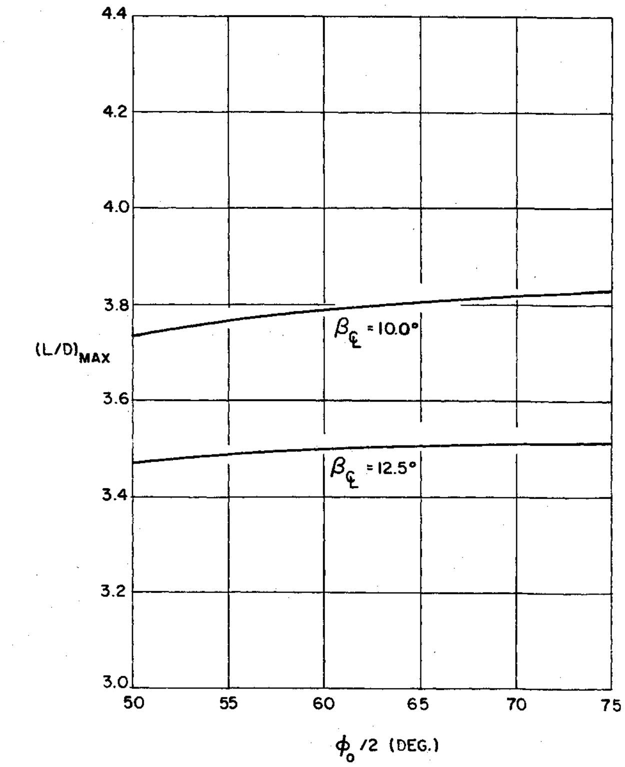

Fig. 4 Maximum L/D vs dihedral angle.

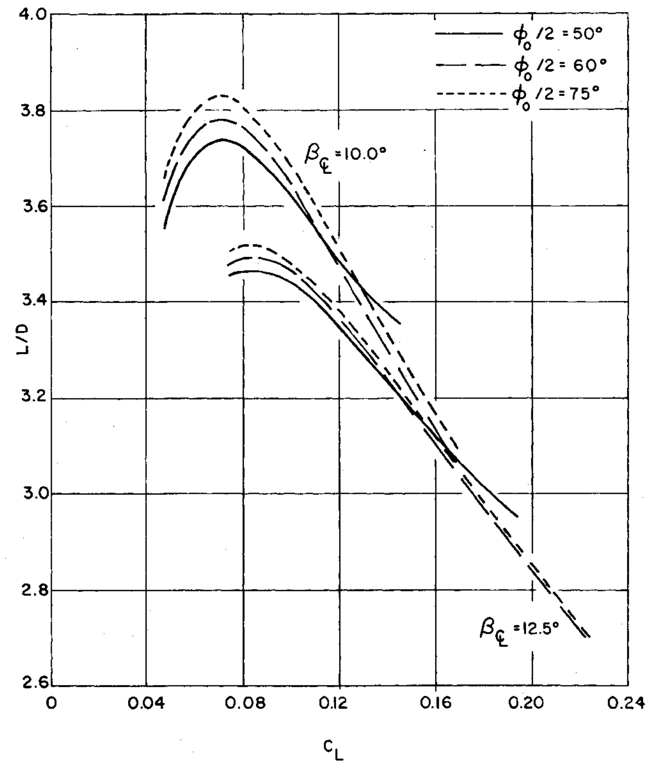

Fig. 6 L/D vs lift coefficient.

$\tau = \frac{2}{3}(\tan \theta_w)\mathrm{AR}^{-1/2}$. Note that rather than specifying the wedge angle θ_w, one states the centerline shock angle, $\beta_L = f(M_\infty, \theta_w)$, this leads to one less iteration. The sweepback angle is found from L_1 and M_∞ and thus a normal Mach number and wedge angle can be calculated. From the normal wedge angle and normal Mach number, one is able to find a normal shock angle, but must iterate to do so. Utilizing these normal conditions, the pressure on the lower surface can be found and hence, the lift and drag of the geometry.

In order to obtain a better estimate of the drag, a viscous correction is made using Eckert's Reference Enalpy technique. This is done by calculating the external condition behind the shock using $M_{n\infty} \cos \alpha$ as a freestream Mach number and passing this through a wedge angle θ_w. These external conditions allow us to calculate a reference temperature, T' and hence ρ' and μ'.

Computer runs were made to compare the over-all effectiveness of this procedure with that of simple wedge theory as applied in the centerline plane. In particular, the flight conditions peculiar

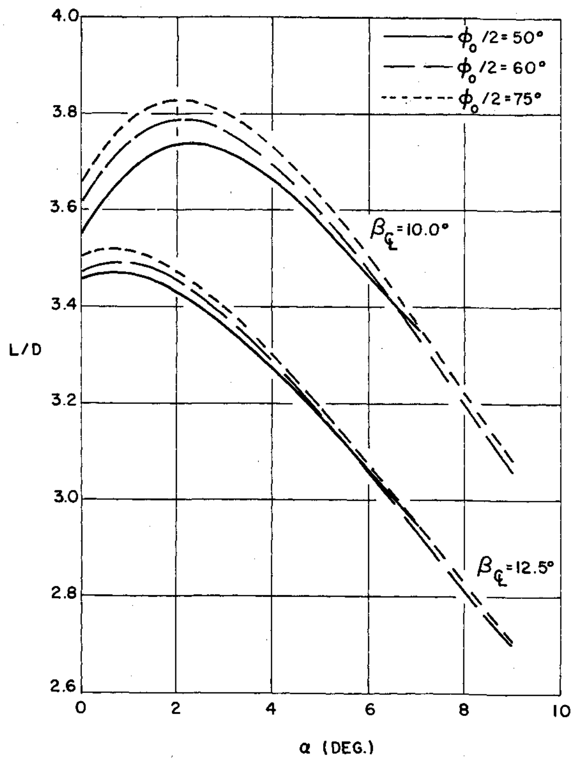

Fig. 5 L/D vs angle of attack.

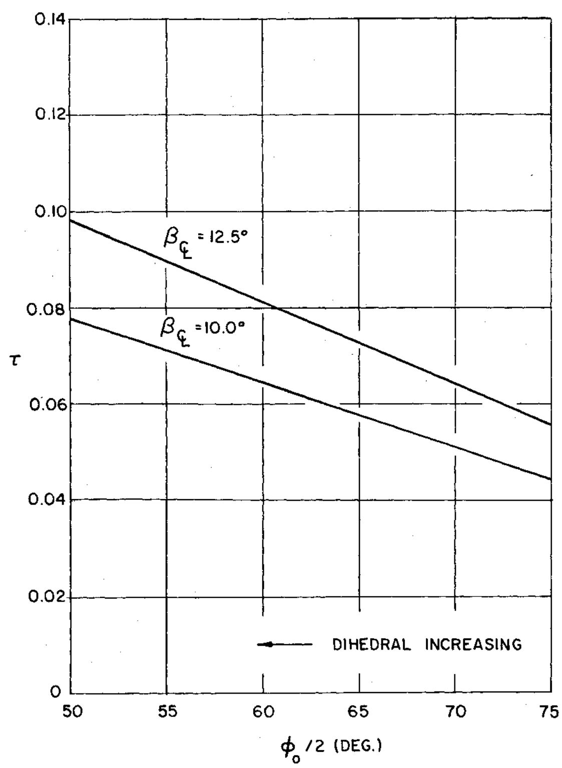

Fig. 7 Volume parameter vs dihedral angle.

to Kipke's waveriders were used so that a proper comparison could be made. Figure 3 shows a typical correlation with some of the available experimental data. It is seen that the agreement is quite good, in particular, the viscous correction on the C_D parameter gives expecially good results. Figures 4–6 depict the influence of body geometry on the lift to drag ratio. It is seen for example, that for a given wedge angle (actually, the shock angle is what is shown), the amount of negative dihedral does not greatly effect the maximum attainable ratio of L/D. The higher (L/D) values are attained with smaller wedge angles, as one might expect. These general trends are also observed in the next two figures. In Fig. 5, L/D is plotted as a function of α. Again, the variation of $\varphi_o/2$ is not particularly significant. It is seen, for example, that β_L plays a more influencial role. Figure 6 shows L/D as a function of the lift coefficient. Again the same trends are observed. While the maximum values of L/D seem to indicate that one should seek a slender wedge with minimum dihedral, Fig. 7 shows the major drawback of this configuration. It is seen that τ, the volume parameter, varies directly with both the dihedral and wedge angles. This effect combines to give the minimum volume coefficient at the maximum (L/D) positions. It therefore, becomes apparent that a tradeoff study of $(L/D)_{max}$ as τ is required to mate a particular configuration to a given set of flight parameters.

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Mach Reflection from Overexpanded Nozzle Flows

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FOR two-dimensional nozzle flows under highly overexpanded conditions, the shock generated from the corner at the nozzle exit cannot be regularly reflected from the centerline of symmetry and Mach reflection generally occurs. Referring to Fig. 1, where a typical Mach reflection from such a nozzle flow situation is depicted, somewhere along the straight incident weak shock I, a triple point T appears. A reflected shock R which may be weak or strong and a Mach stem (MS) which is a curved strong shock are initiated from the triple point along with a slip line S indicating the entropy discontinuity. Early studies of regular and Mach reflection of shocks were carried out by Bleakney and Taub and by Taub et al.[1–3] They were mainly concerned with transient flow cases within the shock tube. Excellent agreement has been obtained between the theoretical and experimental results for regular reflection of shocks. Experimental data also supported the three shock theories for the triple point when the incident wave is strong.

The present investigation is restricted to the study of Mach reflection associated with a steady two-dimensional overexpanded nozzle flow. The experience gained in the study of ejector flowfield[4] suggests that this problem belongs to the category of inviscid interaction between the two streams as long as the center core flow is distinct, and the viscous effects, such as prevailing along the slip-line or along the jet boundary, can only contribute to a modifying influence to the flowfield.

It is thus believed that the shock configurations as well as the accompanying flowfield including the Mach stem height can be established through the consideration that the center core flow should eventually reach a state which is equivalent to choking for a uniform one-dimensional flow. In addition, previous study of shock wave-viscous layer interaction[5] provided the evidence that the shock structure can only be modified within the rarefied flow regime, so that the Rankine-Hugoniot shock relations for air flow can be applied with confidence at the triple point when Mach reflection occurs. Perhaps it may be argued that for high nozzle Mach numbers, the flow would separate away from the corner before Mach reflection of a uniform approaching flow appears. It should be realized that under such conditions, Mach reflection with nonuniform approaching flow probably occurs within the nozzle. This situation can be studied only after the model of Mach reflection with uniform approaching flow has been developed.

The method of consideration discussed above is indeed fruitful. Some of the results obtained are reported here. Figure 2 shows the condition of occurrence of triple point configuration. Since all incident shocks are generated from the nozzle corner, the deflection angle δ_2 must be smaller than the attachment limit δ_{al} corresponding to the nozzle Mach number. δ_{rl} represents the limit for regular reflection which has been reported previously.[1] When the usual criteria of equal pressure and streamline angle across the slip-line are imposed to determine the triple point shock system, it is found that the triple point can occur only within the region bounded by the two curves labelled as δ_{n3} and δ_{n4}. δ_{n3} represents the limiting conditions when the reflected shock is "normal" whereas δ_{n4} corresponds to conditions when the Mach stem shock is "normal" at the triple point. These two curves intersect on the horizontal axis around the value of $M_1 = 1.484$ ($\gamma = 1.4$). In addition, sonic conditions behind the reflected shock at the triple point ($M_3 =$

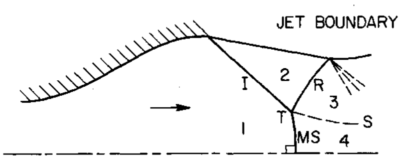

Fig. 1 Mach reflection from overexpanded nozzle flows.

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